A PROOF OF THE TWIN PRIME THEOREM

Paul J. Brinson
Kenmore, New York

OUTLINE OF THE PROOF

1) Let P = any prime, n ≡ the ordinal number of a prime, and P_n = the nth prime.

2) Let P!! or P_n!! ≡ primorial P or the product of the first n primes.

3) A "factor" of an integer, is another integer, other than the first integer, which leaves a remainder of zero when dividing the first integer.

4) Let p = any prim, a prim being any integer from zero though P_n!! which does not have any of the n prime factors of P_n!! as a factor of the prim itself.

5) Let (tp) or (p_1, p_2) be any ordered pair of twin prims from zero through P_n!! +1, of the form (p, p +2), the last pair, (P_n!! – 1, P_n!! + 1), being called the straddle pair.

6) Let [P!!] ≡ the ordered set of integers from zero through P_n!!**,** or the number of them, and [P] be the ordered set of primes from zero through Pn!!, or the number of them.

7) Let [p] or $[p]_n$ ≡ the ordered set of prims from zero through P_n!!**,** or the number of them.

8) Let [tp] or $[tp]_n$ ≡ the ordered set of twin prim pairs from 0 through P_n!!, or the number of them, but also including the straddle pair, (P_n!! - 1, P_n!! + 1).

9) Let $[tp]_x$ ≡ the ordered set of twin prim pairs in [P_n!!] from 0 through x, or the number of them, where x is an integer from zero through P_n

10) Prove that $[tp]_n = (P_1 - 1)(P_2 - 2)\ (P_3 -2)((P_4 - 2)(P_5 - 2)(P_6 - 2)(P_7 – 2)\ldots\ldots(P_n –2)$, the number of twin prim pairs in [P_n!!], provided that the straddle pair, (P_n!! **-** 1, P_n!! + 1) is defined as being "in" [P_n!!], even though the second member of the straddle pair is greater that P_n!!. A symbol for the formula for $[tp]_n$ is "(P_n – 2)!!", and n may increase without upper bound,

11) The ratio of the number of prims in $[P_n!!]$ to $P_n!!$ is called the prim ratio for $P_n!!$ and equals $[p]_n / P_n!!$. The twin prim ratio for $P_n!!$ is $[tp]_n / P_n!!$.

12) $P^2 \equiv {}_{P\text{^}2}$ when P^2 is written as a subscript.

13) $[p]_x/x$ is called the prim ratio through x in $[P_n!!]$, where x is an integer.

14) The prim ratio through $(P_n)^2$ in $[P_n!!] = [p]_{P\text{^}2} / (P_n^2)$.

15) Demonstrate that all of the prims in [Pn!!] less than $(P_n)^2$, except "1", are primes.

16) Prove that where $P_n > 1069$, the number of twin prim and twin prime pairs between P_n and $(P_n)^2$ are always more than "$8P_n$," for all values of P_n, without upper limit.

17) Ergo, there is an infinite number of twin prime pairs.

POINT 1

DISCUSSION OF PRIMS AND PRIMORIAL NUMBERS

1) Let P = any prime, n = the ordinal number of any prime and P_n = the nth prime. ($P_1 = 2$, $P_2 = 3$, $P_3 =5$, $P_4 = 7$, $P_5 = 11$, $P_6 = 13$, ….$P_{10} = 29$ …. ad infinitum.

2) Let $P_n!! = P_1 \times P_2 \times P_3 \times P_4 \times P_5 \times P_6 \times P_7 \times P_8 \ldots. P_{n-1} \times P_n$, the product of the first n primes, and let P!! be identical to $P_n!!$, each being read " P-subn primorial" or "primorial P-subn. ($P_1!! = 2$, $P_2!! = 2 \times 3 = 6$, $P_3!! = 2 \times 3 \times 5 = 30$, $P_4!! = 2 \times 3 \times 4 \times 7 = 210$, $P_5!! = 2 \times 3 \times 5 \times 7 \times 11 = 2310$, $P_6!! = 2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30{,}030$, ….)

3) $[P_n!!]$ represents the ordered set of integers from zero through $P_n!!$, and [P!!] is identical to $[P_n!!]$ when it is understood that P is P_n.

4) A prim is any integer less than $P_n!!$, which does not have any of the prime factors of $P_n!!$ as a factor of itself. "p" represents any prim and $[p]_n$, or [p] with no subscript, represents the ordered set of prims from zero to $P_n!!$, or the number of them, and is known as the prim set for $P_n!!$, or the prim set for P-subn primorial. According to definition, "1" is a prim in each prim set, but the prime factors of $P_n!!$ are not.

5) A nonprim is any integer from zero through $P_n!!$ which does have at least one of the prime factors of $P_n!!$ as a factor of itself. "m" represents any nonprim and $[m]_n$ or [m], without a subscript, represents the ordered set of nonprims from zero through $P_n!!$, or the number of them, and is known as the nonprim set for P-subn primorial. By definition, each of the prime factors of $P_n!!$ is a nonprim. The sum of the number of prims and number of nonprims from zero through $P_n!!$ is $P_n!!$. ($[m] + [p] = P_n!!$)

7) The first five prim sets are as follows:

i) [p] for 2!! = (1), where 2!! = 2

ii) [p] for 3!! = (1, 5), where 3!! = 3x2 = 6.

iii) [p] for 5!! = (1, 7, 11, 13, 17, 19, 23, 29), where 5!! = 5x3x2 =30.

iv) [p] for 7!! = (1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 121, 127, 131, 137, 139, 143, 149, 151, 157, 163, 167, 169, 173, 179, 181, 187, 189, 191, 197, 199, 209) where 7!! = 7x5x3x2 =210.

v) [p] for 11!! = (1, 13, 17, 19, 23 …. 2310 – 13, 2310-1) where 11!! = 2,310.

8) Each prim set, except the first, for a given primorial number, P_n!!**,** may be constructed by taking the prims of the last prior prim set, the prim set for P_{n-1}!!, and adding each of said prims, separately and in numerical order to zero, and also to each of the integral multiples of P_{n-1}!! from "1" to P_n –1 separately and in numerical order, to form what is called the "preprim" set for P_n!!, and then,

9) to complete the construction of the prim set for P_n!! from the preprim set for P_n!!, a set of prims called the "strike set" is stricken from the preprim set, leaving the remaining preprims as the prim set for P_n!! (see illustration infra.).

10) The strike set, [ss], for the preprim set for P_n!!, consists of all of the members of the preprim set in numerical order which have P_n as a factor, or the number of them. The result is that none of the members of the set of prims left after removing the strike set has any of the n prime factors of P_n!! as a factor if itself, and therefore the result is the prim set for P_n!!.

11) The number of members of the strike set for the preprim set of P_n!! equals the number of members of the prim set for P_{n-1}!!.

12) The members of the strike set are constructed by multiplying P_n times each of the

prims of the prim set for P_{n-1}!!, separately, and [ss] = members of the ordered strike set, or the number of them.

13) None of the members of the preprim set for P_n!! have any of the factors of P_{n-1}!! as factors of themselves because each of the members of the preprim set are of the form $N(P_{n-1}!!) + m$, where the first term has all of the prime factors of P_{n-1}!! and the second term has none of the prime factors of P_{n-1}!!. The preprim set for Pn contains all of the integers less than P_n!! that do not have any of the prime factors of P_{n-1} because all of The other integers from zero through P_n!!, the nonpreprims, are of the form $N(P_{n-1}!!) + m$, where the first term and second term do have at least one of the prime factors of P_{n-1}!! in common. This means that to eliminate all of the members of the preprim set for P_n!! which have P_n as a factor, one need only remove the strike set from said preprim set. All of the members of the strike set must be members of the preprim set for P_n!! because they cannot be members of the nonpreprim set for P_n!!, each of which has at least one of the factors of P_{n-1}!!.

14) The first member of the prim set is always "1" and the last is always P_n!! -1.

Tables for prim sets for P_2!! or 3!!, which is 3x2 or 6; P_3!! Or 5!!, which is 5x3x2 or 30!! and P_4!! which is 7x5x3x2 or 210 are expressed below with each leftmost column consisting of the ordered set, from top down, of P_n terms from zero times P_{n-1}!! through all of the integral multiples of said primorial number from "1" to P_{n-1}. The top row consists of the ordered set of prims for P_{n-1}!! The set of numbers at the intersections of the columns and rows constitute the preprim set for P_n!!; the

set of integers which are underlined is the strike set, [ss], from the preprim set; and the integers not underlined constitute the prim set for P_n!!.

PREPRIM AND PRIM SETS FOR P_2!!, OR 3!!

	1
0x2	1
1x2	**3**
2x2	5

(1, 3, 5) is the preprim set for 3!!.

(3) is the strike set for said preprim set.

(1, 5) is the prim set for 3!!.

PREPRIM AND PRIM SETS FOR P_3!!, OR 5!!

	1	5
0X6	1	**5**
1X6	7	11
2X6	13	17
3X6	19	23
4X6	**25**	29

(1, 5, 7, 11, 13, 17,19, 13, 25, 29) is the preprim set for 5!!.

(5, 25) is the strike set for said prim set.

(1, 7, 11, 13, 17, 19, 23, 29) is the prim set for 5!! or, 30.

There are 8 prims in the set and the prim set for 5!! is the largest prim set in which all of the prims, except "1" are primes. There are 22 nonprims from zero through thirty.

PREPRIM AND PRIM SETS FOR P_4!!, OR 7!!

	1	7	11	13	17	19	23	29
0x30	1	**7**	11	13	17	19	23	29
30	31	37	41	43	47	**49**	53	59
60	61	67	71	73	**77**	79	83	89
90	**91**	97	101	103	107	109	113	**119**
120	121	127	131	**133**	137	139	143	149

150	151	157	**161**	163	167	169	173	179
180	181	187	191	193	197	199	**203**	209

The integers in the above table constitute the preprim set for 7!!.

The eight integers (7x1 = 7, 7x7 = 49, 7x11 = 77, 7x13=91. 7x17 = 119, 7x19 =133, 7x 23 = 161, 7x29 = 203) constitute the strike set for 7!!.

The prim set for 7!! consists of the 48 integers in the above chart which are not underlined, or the preprim set minus the strike set.

The set: (1, 11x11= 121, 11x13 = 143, 13x 13 =169) are the prims in the set which consists of prims which are not primes.

Similar tables can be made for all of the subsequent prim and nonprim sets, except that they become so large that they can be handled only by computer or the imagination

Excepting the first prim set, all prim sets are symmetrical about P_n!! / 2, because for each prim "p" in [P_n!!], "P_n-p" is also a prim. Such pair of prims are said to be a "symmetrical" pair of prims and to be complementary one another, as their sum is P_n!!. The prims in a prim set, as a whole, are bilaterally symmetrical, as for each value of p, p/2 + (P!! -p)/2 = P!!/2.

.

The preprim sets and the strike sets are also bilaterally symmetrical. The nonprim sets are also bilaterally symmetrical except that, because zero is neither a prim nor a nonprim, P_n!! has no other nonprim to make a symmetrical pair.

In those systems in which "1" is considered a prime, "1" may be represented by, "P_0." "0" = [P_0!!**]**.

POINT 2

THE FORMULA FOR TH NUMBER OF PRIMS IN A PRIM SET, $[p]_n$, IS:
$[p]_n \equiv (P_n - 1)!! = (P_1 - 1)(P_2 - 1)(P_3 - 1)(P_4 - 1)(P_5 - 1) \ldots\ldots(P_{n-2} - 1)(P_{n-1} - 1)(P_n - 1)$.

By inspection we note that this formula works for the first four primorial numbers:

1) $[p]_1 = 2 - 1 = 1$, “1” being the only prim for 2!!.

2) $[p]_2 = (2 - 1)(3 - 1) = (1)(2) = 2$, the number of prims for 3!!,
the set of prims being (1, 5).

3) $[p]_3 = (2 - 1)(3 - 1)(5 - 1) = (1)(2)(4) = 8$, the number of prims for 5!!,.

the set of prims being (1, 7, 11, 13, 17, 19, 23, 29).

4) $[p]_4 = (2 - 1)(3 - 1)(5 - 1)(7 - 1) = (1)(2)(4)(6) = 48$, the same as the number of prims shown in the prim set for 7!! illustrated in Point 1.The title formula is called the “prim formula”, and, as was shown as in Point 1, the number of prims in a prim set increases, without upper bound as each successive value of “n” increases, without upper bound.

5) To obtain the number of prims in $[p]_5$ or any larger $[p]_n$:

a) Multiply the number of prims in $[p]_4$ by P_5 to obtain $(P_5)([p]_4)$. the number of preprims in the preprim set for P_5, and to obtain the number of preprims in the preprim set for $[p]_n$, multiply the number of prims in $[p]_{n-1}$ by Pn to obtain $(P_n)([p]_{n-1})$.

b) To obtain the number of the number of prims in $[p]_5$ and $[p]_n$, subtract the number of strikes in the strike set for $[p]_5$ and $[p]_n$, $[p]_4$ and $[p_{n-1}]$ from the number of prims in the preprim sets for $[p]_5$ and [p]n respectively, to obtain, $(P_1-1)(P_2-1)(P_3 - 1)(P_4 - 1)(P_5 - 1) \ldots. (P_{n-1} - 1)(P_n - 1)$, which is the title formula and good for all positive values of n without upper bound.

POINT 3

DISCUSSION OF TWIN-PRIMS AND TWIN- PRIM SETS

1) A twin-prim pair is any two prims of the form (p, p + 2), at least one of whose members is in the set of prims for [P_n!!];
2) A prim pair of the form (P_n!! - 1, P_n!! +1) is called the straddle pair for [P_n:], and only the first member of the pair is a member of that set. The first few straddle pairs for the first several prim sets are: (1, 3), (5, 7), (29,31), (209, 211), (2,309, 2,311) … Straddle prim pairs are considered to be "in" [P_n!!];
3) A prim pair of the form (1, P_n!!) is called a polar or cyclical pair for [P_n!!]. Some examples are: (1, 1), (1, 7), (1, 29), (1, 209), and (1, 309);

4) Alternate ways of dealing with the straddle pair are: (a) to consider the straddle pair to be "in" [P_n!! + 1]; (b) to consider the straddling pair for P_n!! to be in [P_{n+1}!!]; or (c) to consider the first member of the straddle pair of prims to be ½ of a twin prim pair and "1" which is in each prim set to be 1/2 of the twin-prim pair; however, for this proof, the treatment of straddle pairs will be considered to be "in" P_n!!.

4) There is one twin-prim pair "in [2!!], (1, 3) (Note that "3", otherwise, is never appears in a prim set.
5) There is one twin-prim pair in [3!!], (5, 7).
6) There are three twin-prim pairs in [5!!]. (11, 13), (17, 19), (29, 31).
7) There are 15 twin-prim pairs in [7!!]: (11, 13). (17, 19). (29, 31), (41, 43), (59, 61), (101, 103), (107, 109), (137, 139), (149, 151), (167, 169) (179, 181), (191, 193), (197, 199), (209, 211).

8) There are 135 twin-prim pairs in [11!!], or [2310], which can be found by inspection.

9) The numbers of twin-prim pairs in each of the above prim sets correspond to the numbers given by the formula for the number of twin-prim pairs in a prim set in the "Outline of the Proof": $[tp] = (P_1 - 1)(P_2 - 2)(P_{3} - 2)(P_4 - 2)(P_5 - 2) \ldots.(P_n - 2) \equiv (P_n - 2)!!$.

10) Each twin-prim set, except the first, for a given primorial number, $P_n!!$, may be constructed by taking the set of twin-prim pairs for the last prior prim set, the twin prim set for $[P_{n-1}!!]$, and adding the members of each of said prim-pairs of said set, separately and in numerical order to zero, and also to each of the integral multiples of$P_{n-1}!!$ from 1 to Pn –1, separately and in numerical order to form what is called the pre-twin-prim set for $[P_n!!]$,which contains P_n times the number of twin-prim pairs in $[P_{n-1}!!]$. To complete the construction of the twin-prim set for $P_n!!$**,** a set of twin-prim pairs called the "twin-prim strike set" is stricken from the pre-prim set for $[P_n!!]$. The number of twin-prim pairs stricken by striking the twin-prims from the pre-twin-prim set is 2 times the number of pre-twin-prim pairs in the twin-prim set for $[P_{n-1}1!!]$, so that the number of twin-prim pairs left in the twin-prim set for $[P_n!!]$ equals $(P_n - 2)$ times the number of twin-prim pairs in the twin-prim pair set for $[P_{n-1}!!]$,

POINT 4

IF “n” IS ANY INTEGER, WITHOUT UPPER LIMIT, THE FORMULA FOR THE NUMBER OF TWIN PRIM PAIRSIN THE PRIM SET FOR [P_n!!] IS: [tp]$_n$ = $(P_1 - 1)(P_2 - 2)(P_3 - 2)(P_4 - 2)(P_5 - 2) \ldots\ldots(P_n - 2) \equiv (P_n - 2)!!$.

Let:

1) “tp” = any twin prime pair;
2) [pt]$_n$ = the ordered set of twin prim pairs in $(P_n - 1)!!$, or the number of them;
3) $(P_n - 2)!! \equiv (P_1 - 1)(P_2 - 2)(P_3 - 2)(P_4 - 2)(P_5 - 2) \ldots\ldots (P_n - 2)$, the title formula for Point 4;
4) The ordered set of prims for P!!, or $(P_n - 1)!!$ is (p1, p2, p3, p4, p5, …pω, in numerical order, with pω being equal to $P_n!! - P_{n+1}$, the $(P_n - 1)!!^{th}$ prim in the set;
5) The “matrix’ for [P_{n+1}!!] =pp-[P_{n+1}];
6) The pre-prim set or matrix for [P_{n+1}] = pp-[P_{n+1}]$_{\}}$, and is the table for the set without C-0.
7) The table for [P_{n+1}!!] consists of $(P_n - 1)!! + 1$ columns labeled from left to right C-0, C-1, C-3, C-4, C-5, ……C-ϖ, with each having a value in the uppermost row of zero, p-1, p-2, p-3, p-4, p-5 ……($P_n!! - 1$), respectively, and P_{n+1} rows labeled from top to bottom R-0, R-1, $-3, R-4 R-5, …… R-$P_{n+!}$ –1 with each having a value in the rightmost column of zero, 1(P_n!!), 2(P_n!!), 3(P_n!!), 4(P!!), 5(P_n!!) ……(P_{n+1-1})(P_n!!), respectively;
8) The numerical value of any cell of the matrix is the sum of two numbers, the number in Row zero of the cell’s row and the number in Column Zero of the cell’s column, as in an ordinary arithmetical addition table, and as is set out in the table for (P_4 - 1)!! With pp-[P_4:] embedded, below;
9) Table for [P_4!!] With pp[P_4:], the preprim set for 7!! embedded:

0	1	**7**	11	13	17	19	23	29
30	31	37	41	43	**47**	**49**	53	59
60	61	67	71	73	77	79	83	89
90	**91**	97	101	103	107	109	113	**119**
120	121	127	**131**	**133**	137	139	143	149
150	151	157	**161**	163	167	169	173	179
180	181	187	191	193	197	199	**203**	209

10) Each of the 8 strikes in the matrix is underlined and in bold face, and there is one and only one strike in each column, except for C-0, that is: (7, 49, 77, 91, 119, 133, 161, and 203).

11) Each of the pairs stricken or subtracted from the above preprim set when a strike is removed is in bold face, and six such pairs are removed: (47, 49), (77, 79), (89, 91), (119,121), (161, 163), or two for each one of the three twin prime pairs in R-0, which consists of the ordered set of prims for P_3!! including the straddle pair, (29, 31) The first member of a straddle pair is considered by previous definition to be 'in" the row of the first member of the pair,, and the second member of the straddle pair in the first column of the next row. The last straddle pair in any preprim set is never stricken because $(P_{n+1})(P_n!! - 1)$, the largest strike, equals $(Pn_{+1})!! - (P_n + 1)$;

12) Two prim pairs in the above preprim and any larger preprim set are stricken for each prim pair in R-0, because a pair is stricken when the first of the prims of the pair

is stricken by the strike in one column, and the second prim of the pair is stricken by the strike in a second non-adjacent column assuming that there is one and only one strike per column in any preprim set.

13) Since there are 21 preprim pairs, and six preprim pairs are stricken, there are 15 twin prim pairs remaining in $(P_4 - 1)!!$ as $7 \times 3 - 2 \times 3 = 5 \times 3 = 15$, in accordance with the formula $(P_4 - 2)(P_3 - 2)(P_2 - 2)(P_1 - 1) = (7 - 2)(5 - 2)(3 - 2)(2 - 1) = 5 \times 3 \times 1 \times 1 = 15$;

14) There is one and only one strike in each column of a preprim set, pp-$[P_{n+!}]$ because:

a. If we let N_a and N_b be two distinct integers, or zero and an integer from 0 to P_n,

b. and"pc", and "pd ", be two distinct prims in R-0 of pp-$[P_{n+1}]$m and "pe" be a different prim in R-0, and

c. $(P_{n+1})(pc)$ and $(P_{n+1})(pd)$ be two distinct strikes in pp-$[P_n + 1]$, then,

d. If we then let: $(N_a)(P_n!!) + pe = (P_{n+1})(pc)$, and

$$(N_b(P_n!!) + pe = (P_{n+!})(p\ d);$$

e. And then subtract the first above equation from the second above equation to obtain: $(N_b - N_a)(P_n!!) = (P_{n+1})(pd - pc)$, in which the right hand side of the equation has a prime factor of (P_{n+1}) and the left hand side of the does not, all of its prime factors being less than $P_{n+.1;}$

15) The last equation is contrary to the Fundamental Theorem of Arithmetic, so that there is one and only one strike in each column of a preprim set.

16) There being one and only one strike in any column of any preprim set, two twin prim pairs from the previous prim set are stricken for each distinct prim pair in the previous prim set, as indicated above, and subtracted from the $(P_{n+1})([p]_n)$ prim pairs so that $[p]_{n+!} = (P_{n+1})([p]_n - 2([p]_n = (P_{n+1} -2)([p]_{n:}$

17) Since $[p]_4 = (P_4 - 2)(P_3 - 2)(P_2 - 2)(P_1 - 1) = (P_4 - 2)!!$,

$[p]_5 = (P_5 - 2)(P_4 - 2)(P_3 - 2)(P_2 - 2)(P_1 - 1) = (P_5 - 2)!!$,

$[p]_6 = (P_6 - 2)(P_5 - 2)(P_4 - 2)(P_3 - 2)(P_2 - 2)(P_1 - 1) = (P_6 - 2)!!$

$[p]n = (P_n - 2)(P_{n-1} - 2)(P_{n-3} - 2) \ldots\ldots(P_3 - 2)(P_2 - 2)(P_1 - 1) = (P_n - 2)!!$

18) Since "n" may be any integer without upper limit, by the convention of mathematical induction, where"n" is without upper limit, $[p]_n = (P_n - 2)!!$, the equation asserted as "POINT 1" is proven for all "n" where "n" is any integer.

POINT 5
EXCEPT FOR “1,”ALL OF THE PRIMS LESS THAN $(Pn)^2$

IN THE PRIM SET FOR $[P_n!!]$ ARE PRIMES.

By definition, none of the prims less than $(P_n)^2$ have factor equal to or less than than P_n, ergo all of prims less than $(P_n)^2$ in [Pn!!] are primes, except for “1.” (E.g. The set of prims less than 7^2 in (P_4!! is 1, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, each of which is prime except “1” . All other integers from zero through 49 are nonprims which have a prime factor of either 2, 3, 5, or 7.

POINT 6
DEFINITIONS AND DISCUSSION OF PRIM AND

NONPRIM RATIOS AND DEVIATIONS

A, Let:

1) $x \equiv$ any integer in $[P_n!!]$;

2) $y \equiv$ a real number from zero through $P_n!!$;

3) $\Delta_1 \equiv$a real number form zero to but, not through 1;

4) $[p]_x \equiv$ the ordered set of prims from zero through x in $P_n!!$, or the number of them;

5) $[m]_x \equiv$ the ordered set of nonprims from zero through x in $P_n!!$ or the number of them;

6) $[p]_n/P_n!! \equiv$ the prim ratio for $[P_n!!]$, $[p]_n$ being the number of prims from zero through $P_n!!$;

7) $[m]_n/P_n!! \equiv$ the nonprim ratio for $[P_n!!]$, $[m]_n$ being the number of nonprims from zero through $P_n!!$;

8) $d_{px} \equiv$ the "prim deviation" in $[P_n!!]$ at x, which is the absolute number of prims which which must be added or subtracted from the number of prims from zero through x to make the resulting prim ratio" at x equal to the prim ratio for $[P_n!!]$ (A synonym for "deviation" is "amplitude.);

9) If the prim ratio at x is greater than the prim ratio for $P_n!!$, the prim ratio at x is said to be positive or positively skewed and $|d_{px}|$ must be subtracted from $[p]_x$ so that the resulting prim ratio at $[p]_x$ equals the prim ratio for $P_n!!$;

10) If the prim ratio at $[p]_x$ is less than the prim ratio for $P_n!!$, the prim ratio is said to be negative or negatively skewed and $|d_{px}|$ must be added to $[p]_x$ to make the resulting prim ratio at x equal to the prim ratio for $P_n!!$;

11) If the nonprim ratio at x is greater than the nonprim ratio for $P_n!!$, the nonprim ratio at x is said to be positive or positively skewed and $|d_{mx}|$ must be subtracted from [m] to make the resulting nonprim ratio at x equal to the nonprim ratio for $P_n!!$;

12) If the nonprim ratio at x is less than the prim ratio for P_n is less than the nonprim ratio for $P_n!!$, the nonprim ratio at x is said to be negative or negatively skewed and $|d_{mx}|$ must be added to $[m]_x$ to make the resulting nonprim ratio at x equal to the nonprim ratio for $P_n!!$;

13) ${}_{maxpos}d_{px}$, ${}_{maxneg}d_{px}$, ${}_{maxpos}d_{mx}$, and ${}_{maxneg}d_{mx}$ equal the maximum positive prim deviation at x, the maximum negative prim deviation at x, the maximum positive nonprim deviation at x, and the maximum, negative nonprim deviation at x respectively, where x may represent different integers, such as y, z and w, in some of the cases;

B.

1) $[p]_x + [m]_x = x$

2) $[p]_x/x + [m]_x/x = 1$

3) $[p]_x/x \pm d_{px}/x = [p]_n/P_n!!$, the sign being plus if the prim ratio at x is negative, and minus if it is positive.

4) $[m]_x/x \pm d_{mx}/x = [m]_n/P_n$, the sign being plus if the nonprim ratio at x is negative , and minus if it is positive.

5) When the prim ratio at x is positively skewed at x, the nonprim ratio is negatively skewed at x, when the prim ratio is negatively skewed at x, the nonprim ratio is positively skewed at x, and the prim and non prim ratios are unskewed at $P_n!!$.

6) At some integer x in $[P_n!!]$, ${}_{maxpos}d_{px} = {}_{maxneg}d_{mx}$, when maxima are measured at integers.

7) At some other integer, y, in $[P_n!!]$, ${}_{maxpos}d_{mx} = {}_{maxneg}d_{px}$, when maxima are measured at integers.

8) The maximum positive d_{px} is always at a prim and the maximum positive d_{mx} is always at a nonprim which is the last nonprim in a series of one or more nonprims. ($P_1!!$ is a special case where the maximum positive dmx is zero.)

9) Whereas , for the purposes of this proof the maximum negative and maximum positive prim and nonprim deviations are always measured at an integer, the deviations themselves need not be an integer, and the maximum negative prim and nonprim deviations, if not required to be measured at an integer, are: for the maximum negative prim deviation, at an infinitesimal before a prim; and for the maximum negative nonprim deviation at an infinitesimal before the first of as series of one or more nonprims.

10) Although not needed for this proof, it can be shown that: (a) The maximum positive prim deviation in $P_n!!$ equals the maximum negative prim deviation, at an integer, plus Δ_1, that is ${}_{maxpos}d_{px} = {}_{maxneg}d_{py} + \Delta_1$, and (b) The maximum negative nonprim deviation at some integer in P_n equals the maximum positive integer at some y plus Δ_1.

(Except for 2!!, the set of prims in $P_n!!$ is bilaterally symmetrical about $P_n!!/2$ and ${}_{maxpos}d_{px}$ is at some integer x in $[P_n!!]$. The interval from x plus an infinitesimal to $P_n!!$, being bilaterally symmetrical about $P_n!!/2$ is the same as the prim distribution from zero through (P_n!!-x), minus an infinitesimal, so that the integer x is not counted twice. This means that to find the maximum negative prim deviation at an integer, it must be measured at (P_n!! –x -1 for the denominator and numerator so that the maximum negative prim deviation is less than the maximum positive deviation by Δ_1. Excepting [2!!] the nonprim set is bilaterally symmetrical about P_n!!, except for the nonprim integer P_n!!, which has no counterpart at zero, and the maximum positivre nonprim deviation is at a nonprim x while the maximum nonprim negative deviation is at an infinitesimal less than the integer (P_n!!-x) to which the integer P_n!!, that must be added, so that ${}_{maxneg}d_{my} = {}_{maxpos}d_{mx} +1 - \Delta_{1.}$ Since Δ_1 is some number less than 1, $1 - \Delta_1$ equals some other Δ_1 In sum, when measured at integers, the maximum positive prim deviation equals the maximum negative prim deviation plus $\Delta_{1,}$ and the maximum negative deviation equals the maximum positive nonprim deviation plus Δ_1, when measured at integers.)

POINT 7

THE PRIM AND NONPRIM DEVIATIONS IN [P_n!!] ARE LESS THAN │.7053│(P

1) The maximum negative prim deviation for [P_n!!] is not more than $\beta_n(P_n) + 1$, where $\beta_n = 1/P_1 + 1/P1xP_2 + 1/P1xP_2xpP_3 + 1/P_1xP_2xP_3xP_4 + 1/P_1xP_2xP_3xP_4xP_5$ +…… + $1/P1xP2xP_3xP_4xP_5x$ ……xP_n + ……. When the coefficient of the series is taken to n terms, it is denominated by "$\beta_{n.}$"

2) "$\beta_n = 1/P_1!! + 1/P_2!! + 1/P_3!! + 1/P_4!! + 1/P_5!!$ + ……1/Pn!! + ……., where "n" can increase without upper bound. In this formulation, each successive term is expressed as a reciprocal of the respective ordered primorial numbers from P_1!! onward.

3) Expressed in numbers, β_n = ½ + 1/2x3 + 1/2x3x5 + 1/2x3x5x7 + 1/2x3x5x7x11 + 1/2x3x5x7x11x13 + 1/2x3x5x7x11x13x17 + ,,,,,,. + $1/P_n$!! +…….

4) For $P_n > 7$, $\beta_n < .7035$, which is about .015 less than (e – 2), "e" being 2. 718281824….

5) To demonstrate that the prim deviations for [P_n!!} are not more than 7.035, first add the integer Pn!! to the positive real number series at zero. (This is consistent with the numbering system in that each integral multiple of a prime factor in [P_n!!] is an integral multiple of a corresponding prime factor in the Pn!! placed a zero. That is, all of the corresponding prime factors in the whole set are congruent to said prime factor. Also, when P_n!! Is added at zero the nonprims in [P_n!!] become bilaterally symmetrical about P_n!! with the extra integer included as a nonprim, and if we consider [P_n!!] to be the cyclic or polar set previously

described as looping back to zero, the use of placing Pn!! at zero is and will be shown to be consistent with the proof of this point.)

6)

POINT 8

THE TWIN PRIM DEVIATIONS FOR [P_n!!] ARE NOT MORE THAN $|P_n|$.

At each of the prims in [P_n!!] the prim and nonprim deviations are not more than $|P_{n,}|$ so that the twin prim deviation at each of the twin prim pairs in [P_n!!] is also not more than $|P_n|$ twin prim pairs, and further, said twin prim deviations are not more than J, or than (e-2.) (See Points 6 & 7) Put another way, there can be at most one twin prim deviation for each prim deviation. (Note that no two twin prim pairs or twin prime pairs share an integer, except for (3,5), and (5,7).

POINT 9

$\phi(P_n)$ IS THE PRIM RATIO FOR [P_n!!] TIMES $(P_n)^2$, EXPRESSED IN MULTIPLES OF P_n AND FOR $P_n \geq 311$, ϕ IS GREATER THAN 30.

1) The prim ratio for [Pn!!] equals:

   (1/2)(2/3)(4/5)((6/7)(10/11)(12/13)(16/17)(18/19)(22/23)…..((P_n – 1)/ P_n),

   which also can be also be represented by, by transposing each of the numerators one place to the left as follows:

   (2/2)(4/3)(6/5)(10/7)(12/11)(16?13)(18/17)(22/19) …….1/$P_{n.}$ Since each term of this expression is greater than 1, except the first, which equals 1, and the last which is 1/Pn. ϕ increases both as P_n or $(Pn)^2$ increase .

2) As examples:

   a) The prim ratio for [19!!] x 19^2 is greater than 2.78(19), so that ϕ for {19!!} is greater than 2.78.

   b) The prim ratio for 311!! Is approximately .096823717, which when multiplied by $(311)^2$ equals more than (30.11) (311), so that ϕ for all primes greater than 311, ϕ is greater than 30.11. (P_{64} = 311))

3) The number of primes from 311 to $(311)^2$, as counted is 9,246, and as calculated for the average number of prims in the same period, (all of which are primes, except"1") the number of primes is 9,365, a difference of approximately 119. Since the maximum prim deviation is less than (.7182818210)311)311 or less than 225, the counted number lies within the number allowed by the prim deviation for the primes subsequent to 311, the number of primes between them and their squares is at least 29(P_n + x).

4) It can be shown that for each

(Quaere: Does ϕ increase without upper bound as P_n increases without upper bound, and, where Pn is sufficiently large, is the prim deviation always positive thereafter?)

POINT 10

$\sigma(P_{n)}$ IS THE TWIN PRIM RATIO FOR $[(P_n -2)!!]$ TIMES $(P_{n)}^2$, EXPRESSED IN MULTIPLES OF P_n, AND FOR $P_n \geq 1069$, $\sigma > 8.954$.

The twin prim ratio for $[(P_n - 2)]$ is $(P_n - 2)!!/P_n$, and is: $(1/1)(1/2)(3/5)(5/7)(9/11)(11/13)((15/17)(17/19)(21/23)\ldots\ldots\ldots((P_n - 2))/P_n)$. By transposing each of the numerators one position to the left, the twin prim ratio can be expressed as:
$(1/1)(3/2)(5/5)(9/7)(11/11)(15/13)(17/17)(21/19)\ldots\ldots\ldots((P_n -2)/P_{n-1}))(1/P_n$. Multiplying this by P_n gives σ and $\sigma(Pn)$ is the average number of twin prim pairs in a consecutive series of $(P_n)^2$ integers in $[(P_n - 2)!!]$.

The calculated twin prim ratio for $P_n = 1069$ is .008376109…. This twin prim ratio times $(1069)^2$ equals (8.954…)(1069), so that σ for 1069 is greater than 8.954, that is $\sigma_{1069} > 8.954$. Since σ increases gradually as P_n increases without limit, σ is always greater than 8.954 when P_n is greater than 1069, and σ minus the twin prim deviation of not more than E_Z is always greater than 8, when P_n is greater than 1069.(Quaere: Does sigma increase without upper bound as P_n increases without upper bound and is the twin prim deviation either always positive or always negative after P_n is sufficiently large.?)

For an example, consider the prime 1007 its square 1,014,049, and the twin prim ratio for 1007 which is .008576576194…. The latter number times 1,014,049 equals 8,890.87 twin prime pairs between 1007 and 1,014,049 as calculated without consideration of the deviation. The counted number of twin primes between 1007 and 1,014,049 is 9,508. The difference between the counted number of pairs and the calculated number of pairs is (9508 – 8,890) or 618, the counted number of pair lies within the allowable deviation of 710 twin prim pairs, which is $€_Z(1007)$ or (,70517943…)(1007), (Note that the twin prime deviation is positive.)

POINT 11

THERE IS AN INFINITE NUMBER OF TWIN PRIMES.

Since, beyond 1069, as P_n increases without limit, there are always at least $8P_n$ twin primes between P_n and its square, by the convention of mathematical induction, there is an infinite number of twin primes!!

ADDENDUM 1

A FORMULA FOR THE NUMBER OF PRIMES FROM ZERO THROUGH $(P_n)^2$ is: $[P]_{p\text{^}2} = \{[(P_n -1)!!] \div Pn!!\} \times (P_n)^2) + (n-1) \pm P_n(\beta)$, THE TERM '(n-1)' BEING THE NUMBER OR PRIMES FROM ZERO THROUGH P_n, MINUS "1", WHICH IS A PRIM BUT NOT A PRIME, AND "Δ' BEING LESS THAN $|1|$.

The above formula follows from the foregoing proof. The formula for the number of primes from zero through $(Pn)^2$ given by the Prime Number Theorem is: $[P]_{P\text{^}2} \approx (P_n)^2/\ln(P_n)^2$. The non-analytical prim generated formula of "Addendum 1" may be a useful adjunct to the Prime Number Theorem.

Since the terms "(n-1)" and "Pn(β) become vanishingly small as $(Pn)^2$ increases without limit, the formula: $\{[(Pn - 1)!!] \div Pn!!\} \times ((Pn)^2)) \approx [P]_{Pn\text{^}2}/ \ln(P_n)^{2,}$ as P_n increases without upper bound. These formulae can be used for approximations of $\ln(P_n)^2$.

ADDENDUM 2

A FORMULA FOR THE NUMBER OF TWIN PRIMES FROM ZERO THROUGH $(P_n)^2$, AS P_n INCREASES WITHOUT UPPER BOUND, IS:

$$[tp]_{p\wedge 2} \approx \{[(P - 2)!!] \div Pn!!\} \text{ X } (P_n)^2.$$

The above follows from the foregoing proof and Addendum 1.

ADDENDUM 3

A FORMULA FOR THE FINE STRUCTURE CONSTANT

Let:

1) P = any prime;

2) n = the ordinal number of a prime;

3) P_n = the nth prime;

4) $P_n!!$ = the product of the first n primes or primorial P_n;

5) Let (Pn – 1)!! = the product of the first n primes with each prime reduced by , or: (2 – 1)(3 – 1)(5 – 1)(7 - 1)(11 – 1))13 – 1)(17 – 1)(19 – 1)(23 – 1)(29 – 1)….(P_n – 1). This is the expression for the number of integers from zero through $P_n!!$, called prims, which do not have any of the prime factors of P_n as a factor. (Note that P_{10} = 29.);

6) α^{-1} = 137.0359990710(96), the reciprocal of the fine structure constant;

7) $B_n \equiv [(P_3!!^2 - P_n!!)(P_{10} – 1)!!] \div P_{10}!! = [(2.3.5)^2 – 2.3.5] \times (2 -1)(3-1)(5 – 1)(7 -1)(11-1)(13 – 1)(17 -1)(19 -1)(23 – 1)(29 – 1) \div$ 2x3x5x7x11x13x17x19x23x29;

8) $B_n = 870 \times [(P_{10} – 1)!! \div P_{10}!!] = 870 \times .1594722310194\ldots$;

9) B_n =137.41490343241359…, rational number;

10) m_p =1836.152627161…, the rest mass of the proton, as measured. with a statistical uncertainty of .0000085;

11) m_n = 1838.68366598 the rest mass of the neutron, as measured, with a statistical uncertainty of .000013

12) $(m_p / m_n)^2$ = .99772488033641.,

13) $B_n(m_p / m_n)^2 = 137.036145129\ldots$;

14) $B_n(m_p / m_n)^2$minus α^{-1} is (137.036145129 – 137..035999070(96) = .0000261…. or approximately 1.8 parts in 10 million, whereas the measured value is for α^{-1} is said to be accurate within .7 parts per billion provided by the formulaic value;

15) $B_n(m_p /m_n)^2 \approx \alpha^{-1.}$

16) The statistical uncertainties for the measured values of m_n and m_p are .0000085 and .00085 respectively, so that the formula "$B_n(m_p / m_n)^2$" is an equality through five significant figures and a possible equality thereafter.

The plausibility of the above formula is suggested by the following:

1) It seems to spring naturally from an attempt to prove the Twin Prime Theorem using the prim system of numbers inspired by the use of Euler's Totient;

2) At least two number theorists have found the prime"29" of use in constructing possible numerical representations of the fine structure constant;

3) A recent measurement of the Fine Structure Constant involving 891 four loop Feynman Diagrams suggests a slight association with the number "870" in the formula;

4) "30" is an important number as it is the largest primorial number for which all of the prims, except "1, are prime";

5) The prim and nonprim sets for a primorial numbers are roughly analogous to the bandings of various spectra of light;

6) The prominence of the number $2(2.3.5)^2$ suggests: $m_p \approx 2(2.3.5)^2 + (2.3)^2 =$ 1836.

7) The fact that the formulaic value for $\alpha^{-1,}$ differs from the measured value by only about 1,8 parts in ten million, which is within the allowable statistical uncertainty for m_p, suggests that more may be at work in this matter than mere coincidence.

ADDENDUM 4

$m_p/m_n \approx (\alpha^{-1}/B_n)^{1/2}$, AND $m_p \approx (1/\alpha B_n)^{1/2} m_n$.

The above formulae are derived by rearranging formulae in Addendum 3, and indicate the relationships among the rest masses of the proton, the neutron, the electron, given as "1", the Fine Structure Constant, and B_n.

By inserting the numbers for some of the respective letters in the above formula for m_p, we obtain: $\mathbf{m_p \approx [((\alpha^{-1})(29!!)) / (29 + 1)(29)(29-1)!!]^{1/2} \times m_n}$, a remarkable result.